Journal of Banking and Financial Economics
2024, 1(21), 54–76
ISSN 2353-6845
DOI: 10.7172/2353-6845.jbfe.2024.1.5

https://press.wz.uw.edu.pl/jbfe

Lead-lag and Volatility Point Change Estimations for Cryptocurrencies

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Received: 5 December 2023 / Revised: 3 September 2024 / Accepted: 16 September 2024 / Published online: 5 November 2024

ABSTRACT

This study investigates the lead-lag relationships and volatility dynamics among four major cryptocurrencies – Bitcoin, Ethereum, Solana, and Polygon – during the turbulent year of 2022. We address three primary research questions: (1) To what extent do lead-lag relationships exist among major cryptocurrencies, and how do they challenge or support the notion of market efficiency in the crypto space? (2) How do volatility change points in different cryptocurrencies relate to each other and to major market events? (3) How can the identification of lead-lag relationships and volatility change points inform cryptocurrency investment strategies and risk management practices?

Using a continuous-time lead-lag estimator and a non-parametric volatility change point detection method, we analysed daily price data for the year 2022. Our findings reveal complex lead-lag dynamics, with Polygon unexpectedly emerging as a leading indicator despite its smaller market capitalisation. This challenges the conventional assumption that larger cryptocurrencies like Bitcoin consistently lead market movements, indicating potential inefficiencies in information transmission within the crypto market.

The volatility change point analysis identifies varying frequencies of volatility shifts across the cryptocurrencies, with Polygon experiencing the most frequent changes (7) and Bitcoin the least (3). We observe both clustering of volatility change points around significant market events and variations reflecting the unique characteristics of each cryptocurrency.

Our results suggest that while the cryptocurrency market shows a high degree of interconnectedness, it also exhibits nuanced dynamics that could be exploited for more effective hedging strategies and improved risk assessment. The study highlights the rapid evolution of the cryptocurrency ecosystem, where technological factors and market-specific events can significantly influence price dynamics and volatility patterns.

This research contributes to the growing body of literature on cryptocurrency market behaviour, offering insights into the complex dynamics of this emerging asset class during a period of significant market stress. Our findings have implications for investors, regulators, and researchers seeking to understand and navigate the rapidly evolving cryptocurrency landscape.

JEL classification: G15, C22, C58, E44

Keywords: estimator, lead-lag effect, volatility point change, quasi-maximum likelihood

1. INTRODUCTION

Cryptocurrencies are digital currencies that rely on blockchain technology and are exchanged among participants without the intervention of a third party such as governments or central banks.

In recent years cryptocurrencies have gained in value and popularity and are used for different reasons such as transactions or investment purposes. This situation has led the crypto market to receive growing attention from policymakers, academics, and investors. Hence, many researchers have analysed cryptocurrencies from different perspectives. Some researchers such as Almeida and Gonçalves (2022); Hasan et al., (2022); Sebastião and Godinho (2020); Bouri et al. (2020); and Wang et al. (2019), focused on the hedge and safe-haven properties in cryptocurrency investments. Others have examined the volatility of the cryptocurrency market (see for example Catania and Ravazzolo (2018); Liu and Serletis (2019); Gupta and Chaudhary (2022); Katsiampa (2019), and Baur and Dimpfl (2018)).

The lead-lag relationships and volatility change points have been widely applied in finance and many other fields. See for instance Chan (1992), Brook et al. (2001), De Jong and Nijman (1997), Kavussanos et al. (2008), Lavielle and Teyssiere (2007), Ross (2013), and Andreou and Ghysels (2004). However, there exist few studies that analysed lead-lag relationships and volatility change points between crypto assets. For example, Sifat et al. (2019) studied the lead-lag relationship between Bitcoin and Ethereum using VECM, Granger Causality, Arma, ARDL, and Wavelet Coherence to detect price leadership between the two cryptocurrencies. In addition, most of these studies focused either on lead-lag relationships or volatility change points for the crypto assets. To the best of our knowledge, our study is the first to investigate the lead-lag relationship and the volatility change points in the year 2022 between four cryptocurrencies: Bitcoin, Ethereum, Solana, and Polygon. Hence, our contribution to the literature is twofold. First, we apply a simple continuous-time process based on the work of Hoffman et al. (2013) to model the lead-lag relationships between the four cryptocurrencies. Then we employ the quasi-maximum likelihood estimator method to identify volatility change points for the four cryptocurrencies.

The reason why we focus on the year 2022 in our analysis is because the crypto market encountered numerous difficulties in 2022. At a macro level, central banks concluded a decade of monetary easing, causing a decline in the prices of high-risk assets. As a developing asset class, cryptocurrencies experienced significant downturns (He et al., 2022). The downward pressure on cryptocurrencies became more intense due to the occurrence of unexpected events like the collapse of Terra-Luna, the bankruptcy of FTX, and the increase in the hacking of many blockchain projects. This drew our interest in assessing the lead-lag relationships and the volatility change points of Bitcoin, Ethereum, Solana, and Polygon, as they are among the largest cryptocurrencies in terms of market capitalisation.

1.1. Theoretical Framework and Research Justification

Our study of lead-lag relationships and volatility change points in the cryptocurrency market is grounded in several key financial theories and concepts that justify its importance and application in this field.

a) Efficient Market Hypothesis (EMH)
The analysis of lead-lag relationships in cryptocurrencies directly tests the semi-strong form of the EMH (Fama, 1970). In an efficient market, price changes in one asset should not consistently predict changes in another. By examining lead-lag relationships, we can assess the degree of market efficiency in the rapidly evolving cryptocurrency ecosystem. This is particularly relevant given the 24/7 nature of cryptocurrency trading and the varying levels of maturity across different cryptocurrencies.

b) Portfolio Theory and Diversification
Markowitz's (1952) Modern Portfolio Theory emphasises the importance of understanding correlations between assets for effective diversification. Our study of lead-lag relationships and volatility change points provides crucial insights into the dynamic correlations between cryptocurrencies. This information is vital for investors seeking to optimise their cryptocurrency portfolios and manage risk effectively in this highly volatile market.

c) Behavioural Finance
The presence of lead-lag relationships may indicate behavioural biases among cryptocurrency investors, such as the attention bias (Barber & Odean, 2008) or the disposition effect (Shefrin & Statman, 1985). By identifying which cryptocurrencies tend to lead or lag, we can gain insights into how information is processed and how investor behaviour might differ across various crypto assets.

d) Market Microstructure Theory
The study of lead-lag relationships in cryptocurrencies contributes to our understanding of market microstructure in digital asset markets. As O'Hara (1995) argues, the way in which trading mechanisms affect price formation is crucial for understanding market behaviour. Our analysis can shed light on how the unique features of cryptocurrency markets, such as decentralised exchanges and 24/7 trading, influence price discovery processes.

1.2. Research Questions and Hypotheses

Based on the above theoretical foundations, we propose the following research questions (RQ) and hypotheses (H):

RQ1: To what extent do lead-lag relationships exist among major cryptocurrencies, and how do they challenge or support the notion of market efficiency in the crypto space?
H1: Significant lead-lag relationships exist among major cryptocurrencies, indicating potential inefficiencies in information transmission within the crypto market.

RQ2: How do volatility change points in different cryptocurrencies relate to each other and to major market events?
H2: Volatility change points in different cryptocurrencies will show some level of clustering around significant market events, but with variations that reflect the unique characteristics of each cryptocurrency.

RQ3: How can the identification of lead-lag relationships and volatility change points inform cryptocurrency investment strategies and risk management practices?
H3: The patterns of lead-lag relationships and volatility changes can be used to develop more effective hedging strategies and improve risk assessment in cryptocurrency portfolios.

1.3. Importance and Practical Applications

Understanding lead-lag relationships and volatility dynamics in the cryptocurrency market is crucial for several reasons:

- For investors, this knowledge can inform more sophisticated trading strategies, potentially allowing for better market timing and risk management.
- For regulators, insights into how shocks propagate through the cryptocurrency ecosystem can aid in developing more effective policies to ensure market stability and protect investors.
- For market designers and cryptocurrency developers, understanding these dynamics can guide improvements in market structure and protocol design to enhance efficiency and stability.
- For academic researchers, our study contributes to the growing body of literature on cryptocurrency market behaviour, providing a foundation for further investigations into the unique characteristics of this emerging asset class.

By addressing these research questions and testing these hypotheses, our study aims to provide a comprehensive understanding of the complex dynamics within the cryptocurrency market. This knowledge is not merely academic but has practical implications for a wide range of stakeholders in the rapidly evolving digital asset ecosystem.

The remainder of the paper is organised as follows. Section 2 discusses the literature review, and Section 3 presents the methodologies. We then present the main results in Section 4 and the conclusion in Section 5.

2. LITERATURE REVIEW

Some pairs of assets share a lead-lag effect which measures the relationship between two variables when the changes in one variable occur before (leads) or after (lags) the changes in the other variable (Hoffman et al., 2013). The lead-lag effect is used to assess the quality of risk management and thus the effective use of the lead-lag effect results in the prediction of future behaviour of stock prices. De Jong and Nijman (1997) apply the proposed estimator for the covariance and correlation to the lead-lag relation between two stock markets' index returns and index future returns. Ito and Sakemoto (2020) investigate the lead-lag relationship in high-frequency data, which showed that a lead-lag effect occurs during significant announcements.

Many assets follow fluctuations of other assets with a small time-lag. Effective use of the lead-lag relationship may lead to the prediction of the future behaviour of stock prices. Different studies suggest different methods used to determine the lead-lag relationships in finance. Sahoo and Kumar (2022) employ the Granger causality test to investigate the lead-lag relationship among three segments of the Indian market. The lead-lag coefficient between two stochastic processes is estimated in Chiba (2019) by utilising fractional Brownian motion. They construct a reliable estimator for the lead-lag parameter, considering a potential convergence rate. To statistically analyse the lead-lag effect, Chiba (2019) proposes estimating the lead-lag parameter between two stochastic processes driven by fractional Brownian motion with a Hurst parameter greater than 0.5. They begin by introducing a regular semi-martingale with a lead-lag parameter ϑ within the range of $(-\delta, \delta)$, where δ is a positive value. Furthermore, they develop a reliable estimator for ϑ by utilising this framework.

Volatility is one of the important measures for many economic and financial applications such as risk management, option pricing, and asset pricing. Volatility measures the rate at which the price of an asset increases or decreases over a certain period. Higher volatility indicates higher risks thus this helps investors in making decisions by estimating fluctuations that may occur in the future. Thus, it is important to track the changes in volatility over different periods, known as time-varying volatility. The analysis of volatility that changes over time (time-varying volatility) in the financial sector has been highly recognised as an important feature. Recently, the focus of many researchers has been on detecting and locating changes in volatility, in particular the shifts in volatility observed in financial time series.

To estimate volatility change points, Kim et al. (2014) consider a series of log returns and propose the Gaussian quasi-maximum likelihood estimator method to detect and locate multiple volatility shifts. The proposed method suggests that the method being discussed is designed to yield consistent and accurate estimates when analysing data. It assumes that the log returns, which can either be dependent or independent, follow a normal distribution. To estimate the number of volatility shifts, a sequential method based on the binary segmentation method is applied. This is a recursive algorithm that is specifically designed to identify volatility shifts and accurately determine the count (number) of shifts for GARCH models. This method involves reducing the negative likelihood by adding a volatility shift to the difference between a point when there is zero volatility shift and the next point when there is a shift. Once the first volatility shift is detected, two subsamples are obtained. This procedure is repeated until no further detections are observed, however, the procedure is slow and time-consuming, thus in this paper we consider the rolling window volatility by setting a window period.

The rolling window volatility can be extended to the change point estimations of cryptocurrencies. To estimate the value at risk and expected shortfall based on the rolling window, Caporale and Zekokh (2019) apply GARCH models to the log returns of four cryptocurrencies. The aim was to choose the best model for modelling volatility for Bitcoin, Ethereum, Litecoin, and Ripple. Analysis shows that using the GARCH models may yield incorrect value at risk and expected shortfall. However, there is a need to understand the nature of volatility in cryptocurrency. The nature of the volatility and interdependence of the cryptocurrencies was identified by analysing the performance of the four cryptocurrencies that are traded mostly in terms of risk and return. Kyriazis et al. (2019) examine the volatility of the three largest digital currencies, i.e., Bitcoin, Ethereum, and Ripple using the GARCH models. It determines the impact of the three highest capitalisation currencies on the other cryptocurrencies using daily data from the period of 1st January 2018 to 16 September 2018, as it was a harsh year for the crypto market. Results show that many digital currencies exhibit a complementary relationship with the three currencies. The study described above does not capture the points at which the volatility changes.

Thus, it is important to estimate the volatility values at different points by making use of estimating methods that estimate parameters of a given probability distribution under the assumption. Ruiz (1994) suggests that the changes in volatility can be modelled using stochastic volatility models. However, most of the popular models are based on the Autoregressive Conditional Heteroskedasticity (ARCH), which suggests a Gaussian distribution. Ruiz (1994) analyses the properties of the quasi-maximum likelihood (QML) estimator of stochastic models based on the Kalman filter, where it shows that for the parameter values often found in the empirical analysis of high-frequency financial time series, the QML estimator outperforms in terms of efficiency. By comparing four different estimators based on the method of moments, results show that parameter values arise when analysing high-frequency financial series. The generalised method of moment estimators is not as efficient as the quasi-maximum likelihood estimators. Moreover, Ozer-Imer and Ozkan (2014) apply change point estimation based on geometric Brownian motion using the change point analysis to investigate the impact of the 2008–2009 financial crises on currencies. By building a relationship between volatility and the crisis, results show that there is an inverse relationship between volatility and crisis.

The existing literature on lead-lag relationships in cryptocurrency markets presents a complex and sometimes contradictory picture. Sifat et al. (2019) employed a multi-method approach including VECM, Granger Causality, and Wavelet Coherence to examine the lead-lag relationship between Bitcoin and Ethereum. Their findings suggested a bidirectional causality, contradicting the conventional wisdom that Bitcoin, as the largest cryptocurrency, should consistently lead the market. This study highlighted the need for more nuanced analyses that consider the evolving nature of cryptocurrency markets. Expanding on this, Goczek and Skliarov (2019) investigated lead-lag relationships across a broader range of cryptocurrencies. Their results indicated that while Bitcoin often led smaller altcoins, the relationships were not stable over time and could reverse during periods of market stress. This temporal instability in lead-lag relationships underscores the challenges in applying traditional financial theories to the highly volatile and rapidly evolving cryptocurrency market.

However, these studies primarily focused on daily or lower frequency data. In contrast, Corbet et al. (2018) utilised high-frequency data to examine intraday lead-lag relationships. Their findings revealed that information transmission in cryptocurrency markets occurs at much shorter intervals than in traditional financial markets, sometimes in the order of minutes or even seconds. This raises important questions about the applicability of conventional market efficiency theories to cryptocurrency markets and suggests the need for more sophisticated, high-frequency analysis tools. While there is a growing body of literature on cryptocurrency volatility, studies specifically addressing volatility change points in this context remain scarce. Existing research, such as Katsiampa (2019), has primarily focused on modelling cryptocurrency volatility using GARCH-type models. However, these approaches often assume a continuous evolution of volatility and may not adequately capture sudden structural changes.

The work of Ardia et al. (2019) represents one of the few attempts to model regime changes in cryptocurrency volatility. Using a Markov-switching GARCH model, they identified distinct volatility regimes in Bitcoin returns. However, their study was limited to Bitcoin and did not explore the potential interconnectedness of volatility regimes across different cryptocurrencies.

This gap in the literature points to a critical need for research that combines the analysis of lead-lag relationships with the identification of volatility change points across multiple cryptocurrencies. Such an approach could provide a more comprehensive understanding of the dynamic risk landscape in cryptocurrency markets. This study aims to help fill this gap.

3. METHODOLOGY

A critical analysis of the methodologies employed in existing studies reveals several limitations. Many studies, including those by Sifat et al. (2019) and Goczek and Skliarov (2019), rely heavily on traditional econometric techniques developed for conventional financial markets. While these methods provide valuable insights, they may not fully capture the unique characteristics of cryptocurrency markets, such as 24/7 trading and the impact of technological factors on price dynamics. Moreover, the majority of studies focus on a limited number of major cryptocurrencies, primarily Bitcoin and Ethereum. This narrow focus fails to account for the growing diversity of the cryptocurrency ecosystem and the potential influence of smaller, more technologically advanced cryptocurrencies on market dynamics.

Another significant limitation in the existing literature is the lack of integration between studies on lead-lag relationships and those on volatility dynamics. This separation has resulted in a fragmented understanding of cryptocurrency market behaviour, where the interplay between price leadership and volatility regimes remains largely unexplored.

To help fill these gaps, the methods (stochastic in nature) employed in this study are presented in the subsections below.

3.1. Quasi-Maximum Likelihood Estimator Method

Maximum likelihood estimation (MLE) is a statistical approach used to estimate the parameters of a given probability distribution using available observed data. MLE entails maximising a likelihood function to establish the point in the parameter space that is most likely to produce the observed data under an assumed statistical model. The maximum likelihood estimate represents the point in the parameter space that yields the highest probability of the observed data. Due to its intuitive and flexible nature, maximum likelihood has emerged as a dominant statistical inference method. When the process is a real Geometric Brownian motion, we can proceed with estimating the parameters using quasi-maximum likelihood estimation, which is a two-stage procedure. In order to increase the performance of the change point estimator in finite samples we are going to use quasi-maximum likelihood function. In the first stage, a 'quasi-likelihood' function is defined to capture the essential features of the true likelihood function while being computationally tractable.

In the second step, the parameters of the model are estimated by maximising the quasi-likelihood function. It is important to note that the method may not always provide the most accurate estimates of the parameters. Before estimating, we briefly overview the quasi-maximum likelihood estimation procedure. Given a sample data $X_n = \left(X_{t_i}\right)_{i=0,1,\dots n}$, where each data point corresponds to a specific time $t_i = i\Delta_n$ such that the interval between consecutive data points, $\Delta_n \to 0$ as $n \to \infty$. When this is the case, the quasi-maximum likelihood estimator utilises an approximation of the true log-likelihood for multidimensional diffusions which is given as

$$\ln\left(X_n, \theta\right) = -\frac{1}{2}\sum_{i=1}^{n}\left\{\log \det\left(\theta_0, X_{t_i}\right) + \frac{1}{\Delta_n}\sum_{i=1}^{n}\theta_0\left[\left(\Delta X_i - \Delta_n a_{i-1}\left(\theta_1\right)\right)^{\otimes 2}\right]\right\} \tag{1}$$

where $\theta = \left(\theta_0, \theta_1\right)$, $\Delta X_i = X_{t_i} - X_{t_{i-1}}$, $a_{i-1}\left(\theta_1\right) = a_{i-1}\left(X_{t_i}, \theta_1\right)$ and the symbol $\otimes$ represents the multiplicative binary operation. Then the quasi-maximum likelihood estimator θ is an estimator that satisfies

$$\theta = \arg \widehat{\min}_{\theta} \log\left(X_n, \theta\right). \tag{2}$$

One of the important properties of the QMLE is that the estimator should be consistent.

Let us now construct our estimator based on the work of (Hoffman et al., 2013). Suppose we have Θ as the parameter space. Let $H = \left(\overline{H}, \overline{\overline{H}}\right]$ is an interval. For a parameter $v \in \Theta$, define the shift interval to be $H_v := \left(\overline{H + v}, \overline{\overline{H}} + v\right]$. For a random interval such that $s \to 1_H\left(s\right)$ the elementary predictable process is given by

$$X(H)_t := \int_0^t 1_H\left(s\right) dX_s. \tag{3}$$

To simplify the notification in equation (2), we abbreviate this as

$$X(H) := X(H)_{T+\delta} = \int_0^{T+\delta} 1_H\left(s\right) dX_s. \tag{4}$$

The definition of the shifted Hayashi-Yoshida covariation contrast is as follows:

$$\bar{v} \to U^n(\bar{v}) := 1_{\bar{v}\geq 0} \sum\nolimits_{I\in\tau,\, J\in\beth,\bar{J}\leq T} X(I)Y(J)1_{\{I\cap J_{-\bar{v}}\neq\emptyset\}} + 1_{\bar{v}<0} \sum\nolimits_{I\in\tau, J\in\beth,\bar{J}\leq T} X(I)Y(J)1_{\{I\cap J_{-\bar{v}}\neq\emptyset\}}.$$

This transformation of $\bar{v}$ to $U^n(\bar{v})$ is a function that calculates the sum of the specific product of $X(I)$ and $Y(J)$ based on the values of $\bar{v}$.

To obtain the estimator we must maximise the contrast $\bar{v} \to U^n(\bar{v})$ over an existing finite grid $G^n \subset \Theta$ where $0 \in G^n$ and the following two conditions under it hold:

- $G^n = 0\left(v_n^{-Y}\right)$, for some $Y > 0$,
- $U_{\bar{v}\in G^n}\left[\bar{v}-\rho_n, \bar{v}+\rho_n\right] \supset v$.

3.2. Lead-Lag Estimation

A martingale is a sequence of random variables for which at a given particular time the conditional expectation of the next value is equal to the present value. A semi-martingale exhibits both deterministic and random behaviour, this implies that it is not always a martingale. In this section, our focus will be on the lead-lag relationship between two semi-martingales. The lead-lag model is a two-dimensional process and is given by:

$$\left(X, \tau_v(Y)\right) = \left(X_T, \tau_v(Y)_t\right) \tag{5}$$

where $\tau_v(Y)_t$ represents the time-shifted of the process Y by a lag parameter τ_v, evaluated at time $v \in [0,T]$. The process X is associated with the realisation X_T at time T. The lead-lag model defines a relationship between the random variable X and a time-shifted process Y based on a lag parameter τ_v. This relationship is described over a time interval from 0 to T, with X having a realisation at time T and $\tau_v(Y)$ being evaluated at each time point t within that interval.
In this study, we use the lead-lag statistical model discussed in (Hoffman et al., 2013). Suppose the generated random observations for the pair of assets (X, Y) are subdivisions of the interval $[0, T+\delta]$ shown below:

$$T^X := \left\{s_{1,n_1} < s_{2,n_1} < \cdots < s_{n_1,n_1}\right\} \tag{6}$$

$$T^Y := \left\{t_{1,n_2} < t_{2,n_2} < \cdots < t_{n_2,n_2}\right\} \tag{7}$$

for X and Y respectively, where $n_1 \neq n_2$. Assume $s_{1,n_1} = t_{1,n_1} = 0$ and $s_{n_1,n_1} = t_{n_2,n_2} = T+\delta$. The sample data are space-equipped in time. Using the historical data set of, $X_s s \in T^X$ or $Y_t, t \in T^Y$ our objective is to estimate the unknown parameter that lies within the range $(-\delta, \delta)$. It is important to note that the (X, Y) process exhibits characteristics of a semi-martingale.

Using the notations from Hayashi-Yoshida we shall describe the properties of the combined sampling scheme given as $T^X \cup T^Y$.

3.3. The Change Point Estimation

Let us first define the important notions that will be used in this section.

3.3.1. Definition. (Iacus, 2011) 'Let $F_0 = \{\emptyset, \Omega\}$ be a trivial σ–algebra. A filtration is defined to be a family $F = \{A_t, t \geq 0\}$ of sub σ–algebras $A_t \subset A$ satisfying $A_0 \subset A_s \subset A_t$ for $0 < s < t$.'

We now define a stochastic basis.

3.3.2. Definition. (Iacus, 2011) 'A probability space equipped with the filtration $\{F_t\}_{t\geq 0}$ of its σ–algebra. F is called a stochastic basis given by $\mathrm{B} = (\Omega, F, \mathrm{F}, P)$.'

Under the assumption that the process of our data generation is an Ito process realised on a stochastic basis $\mathrm{B} = (\Omega, F, \mathrm{F}, P)$ with filtration $\{F_t\}_{t \geq 0}$ satisfying the stochastic integral equation given as

$$X_t = \begin{cases} X_0 + \int_0^t \mu_{i,s} X_{i,s} ds + \int_0^t \theta_0 X_{i,s} dW_{i,s} \\ X_{\tau^*} + \int_{\tau^*}^t \mu_{i,s} X_{i,s} ds + \int_{\tau^*}^t \theta_1 X_{i,s} dW_{i,s} \end{cases} \tag{8}$$

where τ^* is the unknown change point and is to be estimated along with θ_0 and θ_1 such that $\theta_0 < \theta_1$ from the observations of the assets. Given $X_i, i = 0,1,\ldots,n$ the aim of this study is to identify and estimate consistently when a change in the value of the parameter θ occurs. We apply the method proposed in Chapter 2 of (Iacus & Yoshida, 2012).

4. RESULTS AND DISCUSSION

4.1. Statistical Description of the Data

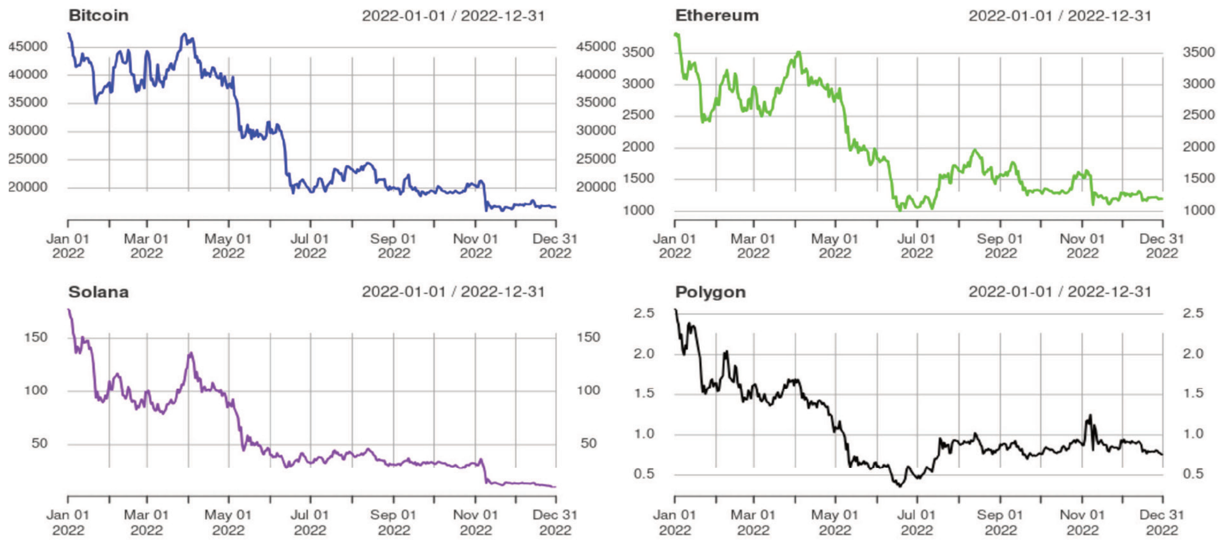
Our dataset was obtained from the historical data from Yahoo Finance. It consists of all trades for the normal year from 1st January to 31st December 2022. The trading happens each day, thus the length of our data is 365. The code used for the estimations is given in Iacus and Yoshida (2018) and was modified to get the required outputs.

Fluctuations in crypto prices are sometimes caused by investors and traders who bet on an ever-increasing price in expecting to make more money. Before estimating the lead-lag effect of the pairs of assets, it is important to first understand the behaviour of their prices during the period 2022. The reason behind this is, when cryptocurrency prices are down, it is difficult for investors to make decisions on whether to sell stocks or hold for a hopeful rebound. Figure 4.1 shows the closing prices for Bitcoin, Ethereum, Solana, and Polygon, respectively. Between January and May, we noticed a gradual decline in the prices of the assets. Prices continued to drop further between the period of June and July for all the pairs of assets, indicating that at least around this period we can expect a change in the volatility for the assets.

The analysis of closing prices for Bitcoin, Ethereum, Solana, and Polygon throughout 2022 is crucial for understanding the context in which our lead-lag and volatility change point estimations are conducted. These price movements reflect the market conditions and major events affecting the cryptocurrency sector during our study period. By visualising these price trends, we can better interpret the lead-lag relationships and volatility changes identified in our subsequent analysis. For instance, periods of significant price drops or increases may correspond to shifts in the lead-lag dynamics or volatility change points, providing valuable insights into the interconnectedness of these cryptocurrencies during market stress.

It appears that all four cryptocurrencies show similar overall trends, suggesting potential co-movement and interdependence. There are noticeable periods of sharp price declines (e.g., between May and June), which could indicate potential volatility change points. Polygon seems to have more pronounced price fluctuations relative to its price level, which might hint at its potential role as a leading indicator. So, we may hypothesise from this observation: Is Polygon the leader among the four cryptocurrencies?

Figure 4.1
Closing Prices for Cryptocurrencies



The prices continued to fluctuate until December. The figures shown above all have a similar pattern in the movement of prices implying that there is a relationship in the movement of these prices, yet it may be important to check how correlated these assets are. This may be verified if we compute the correlation coefficient between the pairs of assets using the famous Pearson correlation. Pearson's correlation coefficient provides insights into how closely the price movements of two cryptocurrencies are related. The correlation only measures the statistical relationship between variables and does not provide insights into the underlying causes of the observed correlation. Thus, it is important to note that a high correlation does not necessarily mean that the cryptocurrencies directly influence each other. The correlation coefficients for the pairs of assets are shown in Table 4.1.

Table 4.1
Correlation coefficients

	Bitcoin	Ethereum	Solana	Polygon
Bitcoin	1.00	0.98	0.95	0.82
Ethereum	0.98	1.00	0.96	0.88
Solana	0.95	0.96	1.00	0.89
Polygon	0.82	0.88	0.89	1.00

Note: The correlation coefficients presented in this table are based on the daily log returns of the respective cryptocurrencies. Log returns were calculated as the natural logarithm of the ratio of closing prices on consecutive trading days, i.e., $R_t = \log\left(\frac{P_t}{P_{t-1}}\right)$ where P_t is the closing price on day t. This approach is commonly used in financial analysis, as it allows for comparison of returns across assets with different price scales and provides a close approximation to percentage changes for small price movements. The use of log returns also helps to normalise the data and reduce the impact of outliers. The correlation coefficients are dimensionless and range from -1 to 1, with 1 indicating perfect positive correlation, -1 indicating perfect negative correlation, and 0 indicating no linear correlation.

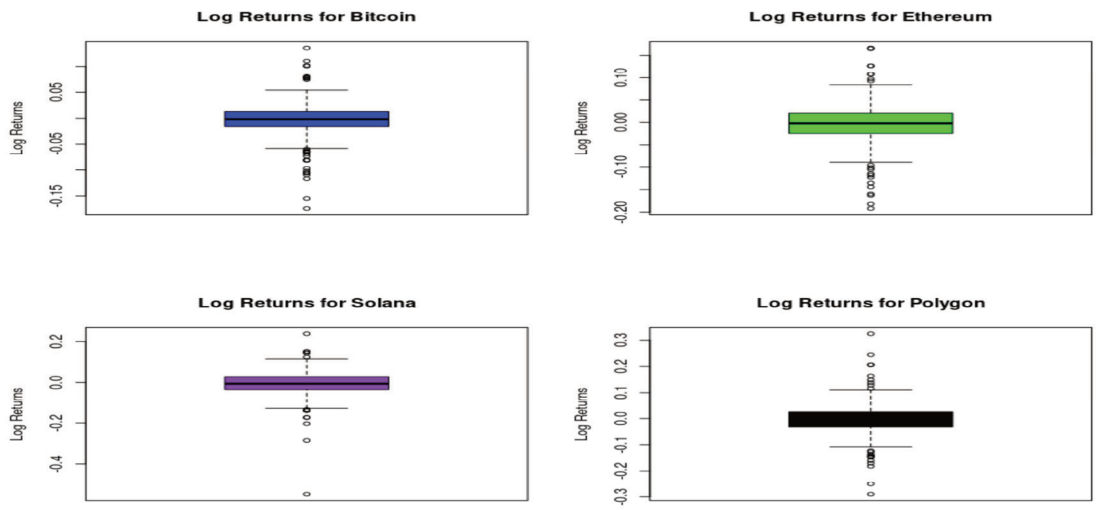
The correlation matrix shows a strong relationship between the assets chosen. This implies that the changes in the prices of one asset will cause the same change in the other asset. A high correlation between two cryptocurrencies can indicate a strong statistical relationship in their price movements. The order from the most strongly related to the least is summarised as follows: Based on the correlation matrix, it appears that Bitcoin and Ethereum, as well as Ethereum and Solana, have the strongest positive correlations among the pairs. This suggests a strong linear relationship in their price movements. The correlations between Bitcoin and Solana, as well as between Solana and Polygon, are also relatively strong. The correlation between Bitcoin and Polygon is somewhat strong, although not as strong as the ones described above. Despite the correlation coefficient, we know that an increase in any one of them may cause an increase in the other at a specified time. This indicates that one must have been the leader and the other must have been the follower. This, however, does not give us enough information as to which asset is leading or following the other, instead merely helping us to understand how the prices of the stocks perform against each other.

Overall, the high correlations between all pairs (ranging from 0.82 to 0.98) suggest strong interconnectedness, which is crucial for lead-lag relationships. The slightly lower correlations of Polygon with the others (0.82 to 0.89) might indicate that it has some independent movements, potentially positioning it as a leading indicator.

The presence of outliers in a dataset may affect the estimation procedure. Thus, it is vital to examine the outliers in the log return prices. This is because the log return prices measure an asset's exponential growth rate. A negative log return implies a drop in the prices of an asset while a positive log return implies an increase in the price. Figure 4.2 shows the box plots of the log returns for the four assets. It is evident that all four assets exhibit outliers. This illustrates extreme market conditions witnessed by these crypto assets and therefore a possible structural break in their volatility.

The box plots (Figure 4.2) of log returns for each cryptocurrency serve a vital purpose in our study. They provide a clear visual representation of the distribution and variability of returns, which is essential for understanding the risk profile of each asset. The presence of outliers, as evident in these plots, indicates extreme market conditions or significant events that may have influenced the lead-lag relationships and volatility patterns that we aim to analyse. These outliers could potentially correspond to the volatility change points that we identify later in our study, thus establishing a connection between the descriptive statistics and our main analytical objectives.

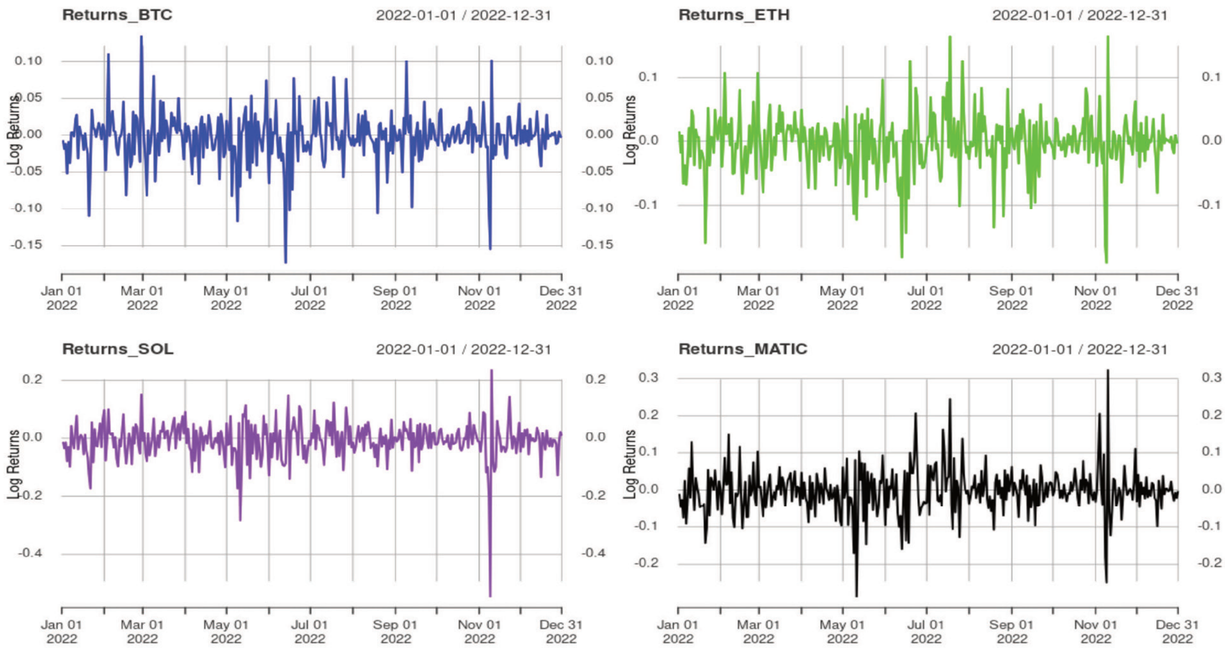
Figure 4.2
Box plot for each cryptocurrency



It is evident that all cryptocurrencies exhibit outliers, indicating periods of extreme returns. Polygon and Solana show a wider range of returns (longer boxes and whiskers), suggesting higher volatility. This higher volatility could make Polygon and Solana more sensitive to market changes, potentially positioning them as leading indicators in the lead-lag relationship.

Crypto assets spot market daily log returns for the sample period of 1 January to 31 December 2022 are shown in Figure 4.3. The log returns for each asset were calculated using the log returns formula $R_t = \log\left(\frac{P_t}{P_{t-1}}\right)$.

Figure 4.3
Daily log-returns for the four Cryptocurrencies for the period 01/01/2022–31/12/2022



The prices of the four cryptocurrencies were transformed into log returns by taking the difference between the two consecutive logarithms of the prices. We can notice that the log returns for Bitcoin were high in March and low in June. The returns for Ethereum were high in July and lowest in November. Solana returns had both the lowest and highest in November, and Polygon's returns were lowest in May and highest in November. We observe from the returns plots that there is volatility clustering indicating that the volatility is not constant over time.

The daily log-returns plots (Figure 4.3) are instrumental in our analysis, as they visually represent the volatility patterns of each cryptocurrency over time. These plots allow us to observe periods of heightened volatility, which are crucial for our study of volatility change points. Moreover, by comparing the log-return patterns across the four cryptocurrencies, we can gain initial insights into potential lead-lag relationships. For instance, similar patterns occurring with slight time delays between different cryptocurrencies might indicate a lead-lag effect, which we formally tested in our subsequent analysis. Thus, these plots provide a foundation for understanding the dynamics we aimed to quantify through our lead-lag and volatility change point estimations.

We can observe that there are visible clusters of high volatility across all cryptocurrencies, but they don't always align perfectly in time. Polygon and Solana appear to have more frequent and larger spikes in returns, which could correspond to more frequent volatility change points. The timing of these volatility clusters might provide initial clues about lead-lag relationships. For instance, if Polygon's volatility spikes consistently precede those of other cryptocurrencies, it might be a leading indicator.

Table 4.2
Descriptive statistics

	Bitcoin	Ethereum	Solana	Polygon
Min	-0.170	-0.190	-0.550	-0.290
Max	0.140	0.170	0.240	0.330
Mean	0.003	-0.003	-0.008	-0.003
Median	-0.002	-0.002	-0.006	-0.003
StD	0.030	0.050	0.060	0.060
Kurtosis	4.610	2.780	14.020	4.690
Skewness	-0.590	-0.370	-1.780	0.210

Note:
Min: The minimum daily return observed for each cryptocurrency.
Max: The maximum daily return observed for each cryptocurrency.
Mean: The average daily return for each cryptocurrency.
Median: The median daily return for each cryptocurrency.
StD: The standard deviation of daily returns for each cryptocurrency, representing the volatility.
Kurtosis: A measure of the 'tailedness' of the distribution of daily returns for each cryptocurrency, with higher values indicating heavier tails and a greater probability of extreme returns.
Skewness: A measure of the asymmetry of the distribution of daily returns for each cryptocurrency. Positive skewness indicates a longer tail on the right side of the distribution, while negative skewness indicates a longer tail on the left side.

The data is more distributed to the left with light tails, which reveals that the log-returns are not normally distributed, and the standard deviation simply indicates that the data is clustered around the mean value. All four cryptocurrencies have high kurtosis, confirming that the data is heavy-tailed and thus contains some outliers. They are all negatively skewed except in the case of Polygon, which exhibits a positive skewness value. Bitcoin and Polygon are approximately symmetric with respect to the skewness values, whereas Solana seems to be highly negatively skewed, while Ethereum is moderately skewed. The value of the standard deviation for all the cryptocurrencies seems small, indicating that the data is clustered around the mean.

The descriptive statistics presented in Table 4.2 offer valuable insights that directly inform our lead-lag and volatility analysis. The measures of central tendency (mean and median) and dispersion (standard deviation) provide a quantitative basis for comparing the return characteristics of the four cryptocurrencies. This comparison is crucial for understanding why certain cryptocurrencies might lead or lag others in price movements. Furthermore, the kurtosis and skewness values indicate the presence of fat tails and asymmetry in the return distributions, which are important considerations in our volatility change point analysis. These statistical properties help explain why we might observe certain patterns in our lead-lag relationships and volatility shifts, thus connecting our descriptive analysis to our main research objectives.

Some of the key observations from Table 4.2 is the following:

(a) Polygon has the highest positive skewness (0.21), indicating more extreme positive returns. This could suggest that Polygon might lead in upward price movements.

(b) Solana has the highest negative skewness (-1.78) and kurtosis (14.02), indicating more extreme negative returns and fatter tails. This suggests Solana might be more reactive to negative market events.

(c) Bitcoin has the lowest standard deviation (0.03), suggesting it might be less volatile and potentially lag behind the others in price movements.

Some preliminary conclusions we can draw from this initial analysis are the following:

1. *Lead-Lag Relationships*: Based on the higher volatility and more extreme returns, Polygon and Solana might be candidates for leading indicators in the lead-lag relationships. Bitcoin, with its lower volatility, might be a lagging indicator.
2. *Volatility Change Points*: The presence of clear volatility clusters in the log-return plots suggests that all cryptocurrencies experienced multiple volatility change points throughout 2022. The more frequent and pronounced spikes in Polygon and Solana suggest they might have more numerous volatility change points.
3. *Market Dynamics*: The year 2022 appears to have been a period of high volatility and significant market stress for all four cryptocurrencies, with several potential regime shifts visible in the price and return plots.

These preliminary findings set the stage for the more rigorous lead-lag and volatility change point analyses that follow in the study, providing context and initial hypotheses to be tested with the more advanced methodologies.

4.2. Lead-lag Results

While our study employs a continuous-time process based on the work of Hoffman et al. (2013) to model lead-lag relationships, it's important to contextualise this approach within the broader landscape of time series analysis. Vector Autoregression (VAR) and Vector Error Correction (VEC) models are widely used alternatives for investigating interdependencies between variables, particularly in the context of short- and long-term dependencies.

VAR models are particularly useful for capturing linear interdependencies among multiple time series and are often applied in analysing the dynamic impact of random disturbances on a system of variables. In the context of cryptocurrency markets, a VAR model could potentially capture how shocks to one cryptocurrency's returns affect the returns of others over subsequent periods. However, VAR models assume a fixed time interval between observations and may not fully capture the continuous-time nature of cryptocurrency trading.

VEC models, on the other hand, are an extension of VAR models that can account for the cointegration between time series. This makes them particularly suitable for analysing long-term equilibrium relationships between variables, alongside short-term dynamics. In the cryptocurrency context, a VEC model could potentially identify long-run relationships between different cryptocurrencies' prices, while also modelling short-term deviations from this equilibrium.

Our chosen methodology, based on Hoffman et al. (2013), offers several advantages in the context of our study. Firstly, it allows for the analysis of lead-lag relationships in a continuous-time setting, which is more representative of the 24/7 nature of cryptocurrency markets. Secondly, it can handle non-synchronous data, which is crucial given the potential for different trading frequencies across various cryptocurrencies. Lastly, this method provides a direct estimate of the lead-lag parameter, offering a clear interpretation of the temporal relationship between assets.

We estimated the lead-lag parameter using the lead-lag matrix for four times series of the same length and the following results were obtained.

Table 4.3
The lead-lag coefficients

	Bitcoin	Ethereum	Solana	Polygon
Bitcoin	0.0000	0.0027	0.0027	-0.0027
Ethereum	-0.0027	0.0000	0.0027	-0.0027
Solana	-0.0027	-0.0027	0.0000	-0.0027
Polygon	0.0027	0.0027	0.0027	0.0000

Note: The coefficients represent the estimated lead or lag in days between the cryptocurrency in the row and the cryptocurrency in the column.

- A **positive** coefficient indicates that the cryptocurrency in the row tends to lead the cryptocurrency in the column. For example, the coefficient of 0.0027 in the Bitcoin row and Ethereum column suggests that Bitcoin's price movements tend to precede Ethereum's price movements by approximately 0.0027 days, on average.
- A **negative** coefficient indicates that the cryptocurrency in the row tends to lag the cryptocurrency in the column. For instance, the coefficient of -0.0027 in the Ethereum row and Bitcoin column suggests that Ethereum's price movements tend to lag behind Bitcoin's price movements by approximately 0.0027 days, on average.
- A coefficient of **zero** indicates no detectable lead-lag relationship between the two cryptocurrencies.

The lead-lag coefficients presented in Table 4.3 provide valuable insights into the dynamic relationships among Bitcoin, Ethereum, Solana, and Polygon during the turbulent year of 2022. These results reveal a complex network of interdependencies that challenge some conventional assumptions about cryptocurrency market dynamics.

Firstly, it's noteworthy that Polygon emerges as the overall leader among the four cryptocurrencies studied. This is evidenced by the positive lead-lag coefficients (0.0027) in its row across all other cryptocurrencies. This finding is particularly interesting given that Polygon is not the largest cryptocurrency by market capitalisation among the four. It suggests that, during the period studied, Polygon's price movements preceded those of even more established cryptocurrencies like Bitcoin and Ethereum. This leadership role could be attributed to Polygon's nature as a layer-2 scaling solution for Ethereum, potentially making it more responsive to changes in network activity and development in the broader cryptocurrency ecosystem.

Bitcoin, often considered the market leader due to its size and first-mover advantage, shows a mixed picture in our analysis. While it leads Ethereum and Solana (both with coefficients of 0.0027), it lags behind Polygon (-0.0027). This partial leadership role aligns with Bitcoin's status as a benchmark for the cryptocurrency market, but also indicates that it may not always be the first to react to market changes, particularly in relation to more specialised or technologically advanced cryptocurrencies, such as Polygon.

Ethereum, despite being the second-largest cryptocurrency by market cap, appears to be more of a follower in this analysis. It lags behind both Bitcoin and Polygon, leading only Solana. This could potentially be explained by Ethereum's role as a platform for decentralised applications and smart contracts. Changes in the broader cryptocurrency ecosystem might first manifest in more specialised tokens (such as Polygon) or in Bitcoin as the market bellwether, before being reflected in Ethereum's price.

Solana emerges as the overall follower in this group, with negative coefficients across all other cryptocurrencies. This is an intriguing finding given Solana's reputation for high transaction speeds and growing ecosystem. It suggests that during 2022, Solana's price movements were generally reactive to changes in the other studied cryptocurrencies.

These lead-lag relationships provide valuable information for investors and market participants. They suggest that, at least during the studied period, monitoring price movements in Polygon could potentially provide early signals of broader market trends. However, it's crucial to note that these relationships are not static and may evolve over time, especially in response to significant market events or changes in the underlying technologies.

Moreover, the small magnitude of the lead-lag coefficients (all 0.0027 or -0.0027) indicates that while these lead-lag relationships exist, they occur over very short time horizons. This aligns with the highly efficient nature of cryptocurrency markets, where information is quickly incorporated into prices across different assets.

It's important to interpret these results in the context of 2022's market conditions. The crypto market faced numerous challenges during this period, including regulatory pressures, the collapse of major projects like Terra/Luna, and macroeconomic factors such as rising interest rates. These events may have influenced the observed lead-lag dynamics, potentially causing shifts in which cryptocurrencies led or lagged market movements.

In conclusion, these lead-lag results provide a nuanced picture of cryptocurrency market dynamics during a tumultuous period. They highlight the importance of looking beyond market capitalisation when considering which cryptocurrencies might lead market movements. Furthermore, they underscore the complex and rapidly evolving nature of relationships within the cryptocurrency ecosystem, where technological capabilities, market sentiment, and external events can all play roles in determining how price movements propagate across different assets.

4.3. Volatility Change Point Results

4.3.1. Rolling Volatility and Change Points

The first asset to experience a change in rolling window volatility was Polygon. The second to experience a change in volatility was Solana, while Bitcoin and Ethereum seem to simultaneously have experienced a change in volatility for the first time. Between days 120 to 150 Polygon experienced a change in volatility twice, whereas in the same period, Solana experienced this change point once. Ethereum, Bitcoin, and Polygon experienced a change in volatility in the period between the 150th and 180th day. Between days 200 and 250, only Bitcoin and Polygon experienced a change, whereafter Polygon alone experienced a change in volatility, in the period between 250 and 300. Lastly, in the period between 300 to 365, the results show that Ethereum, Solana, and Polygon had changes in volatility.

Figure 4.4
Rolling Volatility for Bitcoin

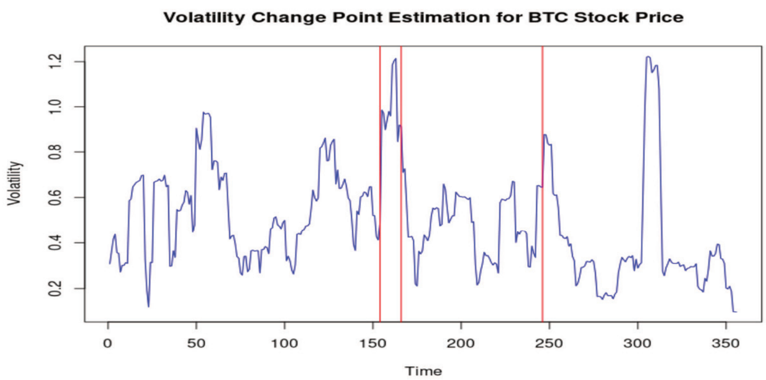


Figure 4.5
Rolling Volatility for Ethereum and Solana

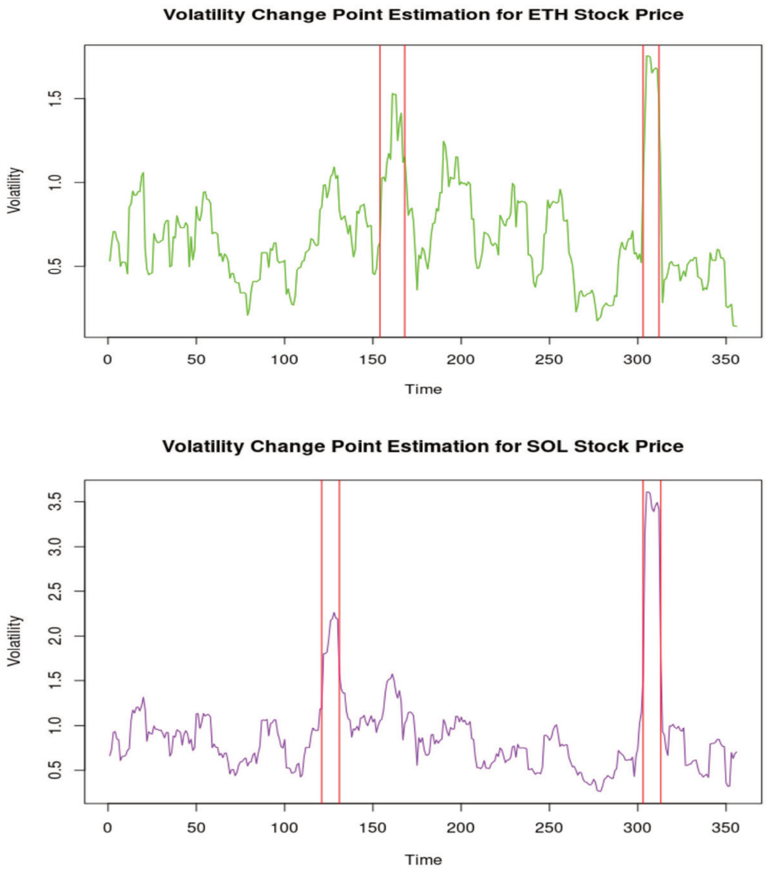
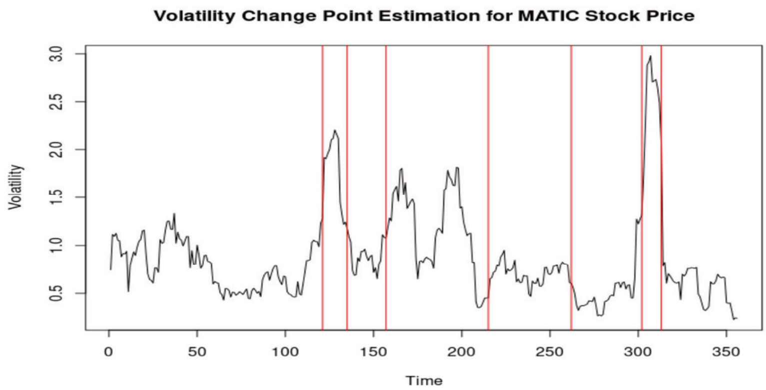


Figure 4.6
Rolling Volatility for Polygon



Note: the horizontal axis represents the number of days.

From the Figures 4.4, 4.5 and 4.6, the points at which the volatility is changing can be related to the price movements discussed in Figure 4.1. It is noticed that when Bitcoin's prices dropped below $20000, there was an increase in volatility, indicating risks. The second time the volatility changed, i.e., when volatility decreased, was during a picking up in the prices. The last change point for Bitcoin shows a period when the volatility was increasing as the stock prices dropped. Similarly, the first time Ethereum experienced an increase in volatility was when the prices were dropping, and the second time was when there was a decrease in volatility as the prices increased slowly. Between October and November, we notice both an increase and a sharp drop in the prices leading to the decrease and increase, respectively. On day 121 around May, we noticed an increase in Polygon's volatility due to the decrease in the price. 10 days later the prices started decreasing, causing an increase in volatility. In summary, an increase the volatility reflects a decrease in asset prices while an increase in volatility reflects a decrease in asset prices. From the above-obtained results, we can conclude that the movement of the prices may give us information on the volatility changes.

4.3.2. Change Points

The figures below show the exact days when these changes occurred in each asset and the number of times it experienced the change points.

Figure 4.7
Change Point estimation for Bitcoin

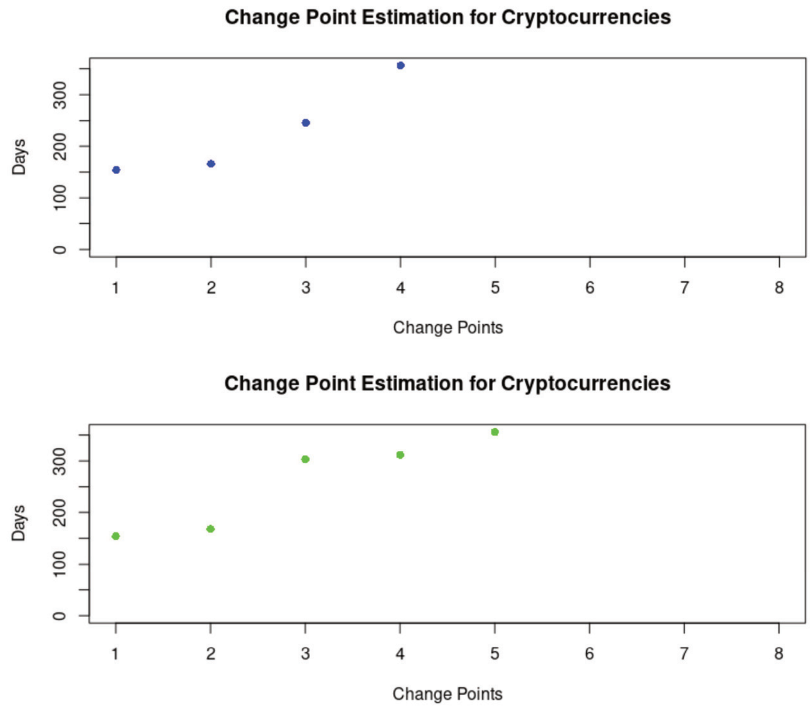


Figure 4.8
Change Point estimation for Solana

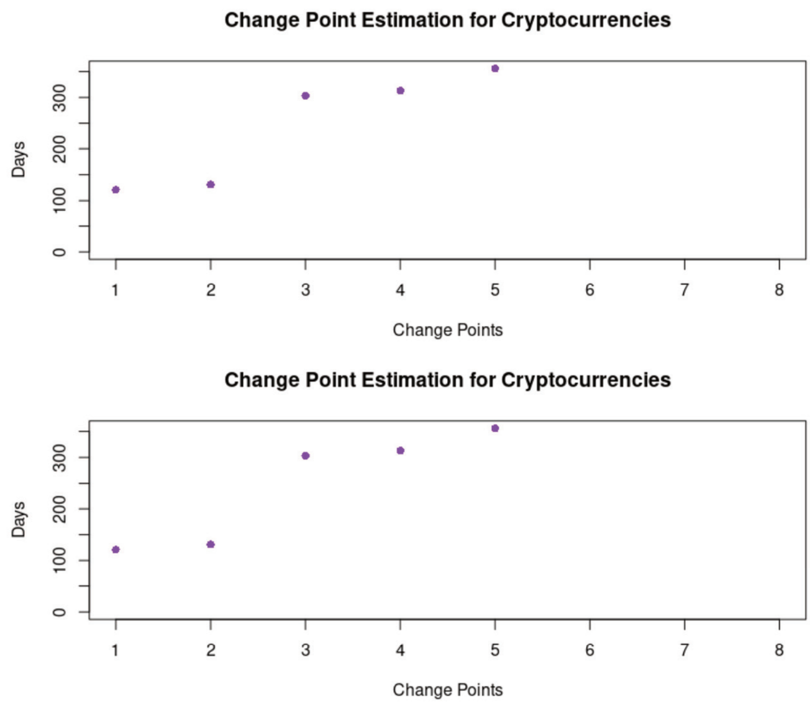


Table 4.4
Summary of the exact days when each cryptocurrency experienced a change in volatility

Number of Times	Bitcoin	Ethereum	Solana	Polygon
1	154	154	121	121
2	166	168	131	135
3	246	303	303	157
4		312	313	215
5				262
6				302
7				313

Table 4.4 presents a comprehensive overview of the volatility change points detected for each of the four cryptocurrencies studied – Bitcoin, Ethereum, Solana, and Polygon – during the year 2022. This data provides crucial insights into the dynamic nature of risk in the cryptocurrency market and allows us to draw several important conclusions.

1. Frequency of Volatility Changes

The most striking observation about Table 4.4 is the varying frequency of volatility changes across the four cryptocurrencies. Polygon stands out with the highest number of change points (7), followed by Ethereum and Solana (4 each), while Bitcoin experienced the fewest changes (3). This disparity in the frequency of volatility shifts suggests that each cryptocurrency responds differently to market forces and external events.

Polygon's frequent volatility changes could be indicative of its higher sensitivity to market conditions or its role as a layer-2 scaling solution, which might make it more responsive to changes in network activity and broader cryptocurrency ecosystem developments. On the other hand, Bitcoin's fewer change points might reflect its status as a more established asset, potentially with a more stable investor base and less susceptibility to short-term market fluctuations.

2. Timing of Initial Volatility Changes

Another crucial observation is the timing of the first volatility change point for each cryptocurrency. Both Polygon and Solana experienced their first change on day 121 (approximately early May), while Bitcoin and Ethereum didn't see a change until day 154 (early June). This suggests that Polygon and Solana may have been quicker to react to changing market conditions in the first half of 2022.

This earlier reaction in Polygon and Solana could be related to their nature as newer, more technologically focused projects, which might make them more sensitive to shifts in market sentiment or technological developments. The delayed response in Bitcoin and Ethereum might reflect their larger market caps and more diverse user bases, which could provide some initial buffer against market volatility.

3. Clustering of Volatility Changes

We can observe some clustering of volatility changes across the cryptocurrencies. For instance, all four cryptocurrencies experienced a change point between days 154 and 168. This clustering suggests the presence of significant market-wide events or conditions during this period (early to mid-June) that affected the entire cryptocurrency ecosystem.

Similarly, there's another cluster of change points across Ethereum, Solana, and Polygon between days 302 and 313 (late October to early November). This could indicate another period of market-wide stress or significant events affecting most of the cryptocurrency market.

4. Unique Patterns

Each cryptocurrency also shows some unique patterns in its volatility change points:
(a) Bitcoin's changes are more spread out, with significant gaps between each change point (154, 166, 246).
(b) Ethereum shows two distinct clusters of change points (154/168 and 303/312).
(c) Solana has a mix of closely spaced and widely spaced change points (121/131 and 303/313).
(d) Polygon shows the most complex pattern, with some closely spaced changes (121/135, 302/313) and some more isolated ones (157, 215, 262).

These unique patterns suggest that while there are market-wide factors affecting all cryptocurrencies, each also responds to specific factors related to its own ecosystem, technology, or user base.

5. Late-Year Volatility Changes

It's noteworthy that Ethereum, Solana, and Polygon all experienced volatility changes late in the year (days 302–313), while Bitcoin did not. This could suggest that these cryptocurrencies were more affected by late-year events in the crypto market (such as the FTX collapse in November) than Bitcoin.

6. Implications for Risk Management

The varying frequency and timing of volatility changes across these cryptocurrencies have important implications for risk management and portfolio diversification. The fact that these assets experience volatility changes at different times suggests that they might offer diversification benefits, as they don't all become more volatile simultaneously.

However, the presence of some clustered change points also indicates that there are periods when diversification benefits might be reduced, as multiple cryptocurrencies experience volatility shifts together.

7. Relation to Market Events

While a detailed event study is beyond the scope of this analysis, it's worth noting that some of these change points likely correspond to significant events in the cryptocurrency market during 2022. For example, the cluster of change points in early to mid-June might be related to the aftermath of the Terra/LUNA collapse in May. The late-year change points in Ethereum, Solana, and Polygon could be connected to the FTX exchange collapse in November.

In conclusion, the volatility change point analysis reveals a complex and dynamic risk landscape in the cryptocurrency market. The varying frequency and timing of these change points across different cryptocurrencies underscore the importance of treating each cryptocurrency as a unique asset with its own risk characteristics, rather than viewing the cryptocurrency market as a homogeneous entity. This analysis provides valuable insights for risk management, portfolio construction, and understanding the evolving nature of the cryptocurrency market during a particularly turbulent year

4.4. Discussion and implications of the results

In terms of market dominance bitcoin has a percentage of 47.2 while Ethereum has a percentage of 19.1 and the market capitalisation for the four cryptocurrencies are ranked as follows:

(1) Bitcoin
(2) Ethereum
(3) Polygon
(4) Solana

Throughout 2022, the crypto market witnessed multiple attacks on blockchain projects. Early in the year, Polygon experienced attacks on a project called Superfluid via the CTX exploit. Approximately six months later, Polygon suffered further breaches, including the price oracle attack and flash loan re-entrancy attack, leading to losses of millions of dollars.

On March 23, 2022, a collateral validation exploits on the Cashio project on Solana resulted in a loss of about $48 million. Ethereum projects faced various threats as well, such as private key compromise attacks, exploits due to a lack of authentic input, and mistakes in migration functions. Bitcoin was not exempt, experiencing its own private key compromise attacks.

Detailed information on these and many other attacks from 2022 is available on the DefiLlama website. Additionally, the crypto market was shaken by the bankruptcy of FTX (FTT), which reportedly lost about $550 million due to a hack. Furthermore, Ronin, an Ethereum sidechain, was hacked through its multi-chain bridge, resulting in losses of hundreds of millions of dollars. More insights into these incidents and other relevant factors can be found on Coinsbench.

The findings from our lead-lag analysis and volatility change point estimation provide valuable insights into the dynamics of the cryptocurrency market during the turbulent year of 2022. These results have several important implications for investors, policymakers, and researchers.

Our lead-lag analysis reveals a complex network of relationships among the studied cryptocurrencies, with Polygon emerging as an unexpected leader. This challenges the conventional wisdom that larger, more established cryptocurrencies such as Bitcoin always lead market movements. The short time horizons of these lead-lag relationships (as evidenced by the small magnitude of the coefficients) suggest that the cryptocurrency market is highly efficient in terms of information transmission. However, the existence of these relationships also indicates that the market is not perfectly efficient, offering potential opportunities for sophisticated traders to capitalise on these brief lead-lag dynamics.

The varying frequency and timing of volatility change points across different cryptocurrencies have significant implications for portfolio diversification and risk management. While all cryptocurrencies studied showed multiple volatility change points, the lack of perfect synchronisation in these changes suggests potential diversification benefits. However, the presence of some clustered change points also indicates periods when these diversification benefits might be reduced. This underscores the need for dynamic risk management strategies in cryptocurrency portfolios, capable of adapting to rapidly changing market conditions.

The fact that Bitcoin experienced the fewest volatility change points among the studied cryptocurrencies might be interpreted as a sign of its relative maturity and stability. This could have implications for Bitcoin's role as a potential store of value or 'digital gold'. However, the more frequent volatility changes in newer cryptocurrencies such as Polygon and Solana highlight the still-evolving nature of the broader cryptocurrency ecosystem. This disparity in volatility behaviour between established and newer cryptocurrencies presents both challenges and opportunities for market participants.

The high frequency of volatility changes, particularly in some cryptocurrencies, underscores the volatile nature of these assets. This volatility, combined with the complex lead-lag relationships, may raise concerns for regulators about market stability and investor protection. Our findings could inform regulatory approaches, potentially supporting arguments for more robust risk disclosure requirements or the need for sophisticated risk management tools in cryptocurrency trading platforms.

The emergence of Polygon as a leader in our lead-lag analysis, despite its smaller market capitalisation, suggests that technological factors play a crucial role in cryptocurrency market dynamics. As a layer-2 scaling solution for Ethereum, Polygon's leadership might indicate that market participants are highly responsive to technological advancements and scalability solutions. This implies that investors and researchers should pay close attention to technological developments when analysing cryptocurrency market trends.

The clustering of volatility change points around certain periods (e.g., early June and late October/early November) likely corresponds to significant market events in 2022, such as the Terra/LUNA collapse and the FTX exchange failure. This sensitivity to major events highlights the importance of monitoring both market-specific and macroeconomic factors when assessing cryptocurrency risks and returns.

The identified lead-lag relationships, although occurring over short time horizons, could inform the development of high-frequency trading strategies. However, the rapid nature of these relationships and the frequent changes in volatility underscore the challenges in implementing such strategies and the need for sophisticated real-time analysis tools.

Our findings open up several avenues for future research. These include investigating the stability of lead-lag relationships over longer time periods, exploring the factors that contribute to a cryptocurrency's role as a leader or follower, and examining how volatility change points relate to specific market events or broader economic indicators.

In conclusion, our results paint a picture of a highly dynamic and interconnected cryptocurrency market. The complex lead-lag relationships and frequent volatility changes highlight both the risks and opportunities present in this market. As the cryptocurrency ecosystem continues to evolve, understanding these dynamics will be crucial for all stakeholders, from individual investors to regulatory bodies. Our findings underscore the need for sophisticated, adaptive approaches to cryptocurrency investment, risk management, and market analysis.

4.5. Evaluation of Research Hypotheses and Key Findings

H1: *Lead-lag relationships exist among major cryptocurrencies, indicating potential inefficiencies in information transmission within the crypto market.*

This hypothesis is partially supported by the findings. The study revealed lead-lag relationships among the four cryptocurrencies examined (Bitcoin, Ethereum, Solana, and Polygon). Notably, Polygon emerged as an overall leader, with positive lead-lag coefficients in relation to all other cryptocurrencies. This suggests some level of inefficiency in information transmission, as price movements in Polygon tended to precede those in other cryptocurrencies, including the more well-established Bitcoin and Ethereum.

However, the small magnitude of the lead-lag coefficients (all 0.0027 or -0.0027) indicates that these relationships occur over very short time horizons. This suggests that while there are inefficiencies, the market is still relatively quick in transmitting information across different cryptocurrencies. Therefore, we can conclude that there are indeed lead-lag relationships, pointing to some inefficiencies, but that the rapid nature of these relationships also indicates a high degree of market efficiency.

H2: *Volatility change points in different cryptocurrencies will show some level of clustering around significant market events, but with variations that reflect the unique characteristics of each cryptocurrency.*

This hypothesis is strongly supported by the findings. The volatility change point analysis revealed both clustering and variation across the four cryptocurrencies. There were notable periods where multiple cryptocurrencies experienced volatility change points around the same time. For example, all four cryptocurrencies had a change point between days 154 and 168, likely corresponding to significant market-wide events. Despite this clustering, there were clear differences in the frequency and timing of volatility change points across cryptocurrencies. Polygon experienced the most change points (7) and Bitcoin had the least (3). Solana and Ethereum each had 4 change points, but not always coinciding with each other.

These variations indeed seem to reflect the unique characteristics of each cryptocurrency. For instance, Bitcoin's fewer change points might reflect its status as a more established asset, while Polygon's frequent changes could be due to its nature as a layer-2 scaling solution, making it more sensitive to ecosystem developments.

H3: *The patterns of lead-lag relationships and volatility changes can be used to develop more effective hedging strategies and improve risk assessment in cryptocurrency portfolios.*

The findings provide strong support for this hypothesis, though direct testing of hedging strategies was beyond the scope of the study. The results offer several insights that could be valuable for developing hedging strategies and improving risk assessment: (a) the identification of Polygon as a leading indicator suggests that monitoring its price movements could potentially provide early signals for broader market trends, (b) the varying frequency of volatility change points across different cryptocurrencies indicates that they have different risk profiles; this information could be used to balance risk in a cryptocurrency portfolio, (c) the presence of both clustered and non-clustered volatility change points suggests periods of both correlated and uncorrelated risks across cryptocurrencies. This knowledge could be used to adjust hedging strategies dynamically.

The short-term nature of the lead-lag relationships implies that any hedging strategies based on these relationships would need to be implemented with high-frequency trading capabilities.

While the study doesn't directly test hedging strategies, the detailed insights into the dynamic relationships between these cryptocurrencies provide a strong foundation for developing more sophisticated risk management approaches.

4.6. Limitations and Future Research

One limitation of our study is the focus on a single year, 2022, which was characterised by significant market stress and volatility in the cryptocurrency sector. While this period provides valuable insights into market dynamics during turbulent times, it may not be representative of cryptocurrency market behaviour under more stable conditions. Future research should extend this analysis to include multiple years, including periods of both market stress and relative stability. This comparative approach would allow for a more robust assessment of whether the lead-lag relationships and volatility change points identified in our study are consistent across different market conditions or are specific to periods of high volatility.

For instance, analysing the years 2020 (pre-COVID bull market), 2021 (crypto market peak), and 2023 (post-FTX collapse recovery) alongside our 2022 data could provide a more comprehensive picture of how these dynamics evolve over time and across different market cycles. Such a multi-year analysis could reveal whether Polygon's leadership role in 2022 was a temporary phenomenon related to specific market conditions or part of a longer-term trend in the evolving cryptocurrency ecosystem. Moreover, comparing the frequency and timing of volatility change points across different years could offer insights into how the maturation of the cryptocurrency market affects its volatility characteristics. It could also help identify whether certain cryptocurrencies consistently experience more frequent volatility changes or if this varies depending on market conditions.

This multi-year comparative analysis would not only enhance the robustness of our findings but also potentially uncover long-term trends and cyclical patterns in cryptocurrency market dynamics. Such insights would be invaluable for investors, regulators, and researchers seeking to understand and navigate the complex and rapidly evolving cryptocurrency landscape.

Another limitation of this study is the absence of formal statistical significance tests and confidence intervals for our lead-lag and volatility change point estimations. While the methods employed provide valuable insights into the dynamics of the cryptocurrency market, the lack of these statistical measures means that we cannot rigorously quantify the precision and reliability of our point estimates. This limitation arises from the complexity of the estimation procedures used and the challenges inherent in deriving analytical expressions for the sampling distributions of our estimators. We acknowledge that incorporating formal statistical significance tests and confidence intervals would strengthen the robustness of our findings, and we aim to address this limitation in our future research by exploring alternative estimation techniques or developing appropriate simulation-based approaches for inference.

5. CONCLUSION

We explored the lead-lag estimations, and volatility change points for cryptocurrencies during a difficult year for the crypto industry. First started by estimating the lead-lag relationships that exist among the four assets using the approach discussed by Hoffman et al. (2013). By the lead-lag estimated, this study examined that among the four cryptocurrencies, Polygon was the leader. This, however, did not align with our expectations, thus we introduced the change point estimations as the hypothesis testing procedure for the lead-lag results. The change points indicated both the points at which the risks were higher (increasing volatility) and points that were best for investors to make decisions based on possible fluctuations that may happen in the future. From the analysis, Polygon was found to experience a change in volatility first, and changed multiple times more than the other assets, indicating that it was the overall leader during the eventful year that 2022 had been for the cryptocurrency market.

Using daily data to estimate the lead-lag and volatility change point does not give accurate information to examine the leader or follower of the pair of assets (*X, Y)* as some changes happen on the same day. Thus, it may be important to consider intra-day data, this is left for future work.

This study was carried out based on the specific period when it was a bad time for the crypto market. A future study may be carried out over a calm period so that we estimate the lead-lag and change point estimations. Since the Geometric Brownian motion assumes a constant volatility, we may as well consider using the Garch model, which does not assume a constant volatility.

The study provides substantial support for all three hypotheses, offering valuable insights into the efficiency of information transmission in the cryptocurrency market, the nature of volatility dynamics across different cryptocurrencies, and the potential for using these insights in portfolio management and risk assessment. These findings contribute significantly to our understanding of cryptocurrency market behaviour and open up numerous avenues for further research and practical application.

It's worth noting that our approach focuses primarily on the contemporaneous and near-term lead-lag relationships, potentially at the expense of capturing longer-term dependencies that VAR/VEC models might reveal. Future research could benefit from a comparative analysis, applying both our chosen method and VAR/VEC models to the same dataset. This could provide a more comprehensive understanding of both short-term lead-lag dynamics and longer-term equilibrium relationships in the cryptocurrency market.

Funding

The cost of editing selected articles published in the Journal of Banking and Financial Economics *in the 2022– 2024 is covered by funding under the program "Development of Scientific Journals" of the Ministry of Education and Science under agreement No. RCN/SN/0321/2021/1. Task title: "Verification and correction of scientific articles and their abstracts". Funding value: 21 197,00 PLN; The task consists of professional editing of articles published in English.*

Declaration of Conflicting Interests

The author declared no potential conflicts of interest with respect to the research, authorship, and publication of the article.

Declaration About the Scope of AI Utilisation

The authors did not use an AI tool in the preparation of the article.

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